REVISING THE INDISPENSABILITY ARGUMENT

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ABSTRACT: Pragmatists have tried to account for the truth of mathematics by way of making it dependent on its success in the applied sciences, the so-called "indispensability argument" proposed first by C. S. Peirce. The key line of attack against indispensability is that it leads to an overblown ontology. It was contended that we can usefully reformulate the indispensability argument in a way that avoids the pit-fall of assenting to the mind-independent existence of abstract objects. In so doing, pragmatists' oft-used tack to deal with the exact sciences is rendered plausible. Some consequences for the exact sciences of the revised indispensability argument were discussed.

Keywords: indispensability argument; mathematical realism; pragmatism; abstract objects

Truth, from the pragmatist point of view, is what works within the totality of the collective enterprise of science. The idea is captured by C. S. Peirce's notion of "abduction," which could be formulated thus: If our best scientific theories of *q*, presupposes the existence of *p*, then observations of *q* gives us good reason to believe *p*.[1] The indispensability argument, roughly put, is the idea that mathematics is true because it is indispensable to scientific descriptions, which are already taken to be so.[2]

The indispensability argument has been attacked for a variety of reasons (some of which I consider in § 3), but the thorny issue remains inferring the mind-independent existence of abstract objects, like numbers. Ontology is considered over-blown if it requires positing the mind-independent existence of objects that serve no useful epistemological end (and it is not clear how they ever could). My purpose is to reformulate the indispensability argument in a way that avoids the pit-fall of assenting to the mind-independent existence of abstract objects.

I shall proceed as follows. The first half of the essay is exegetical, and the second portion develops my ideas. In the first two sections, Quine's and Putnam's reasons for advocating the indispensability argument are considered. In the third section, reasons to reject the indispensability argument are criticized. In the final section, a revised version of the indispensability argument is defended.

1. Quine

Foundational epistemology, according to Quine, attempts to justify knowledge on a model akin to an axiomatic system like that of Euclid. In the foundations of mathematics, for example, he distinguishes the conceptual from the doctrinal; the former concerns meaning (clarifying and defining concepts) and the latter concerns truth (establishing laws by proving them) (Quine 1969, 69).[3]

Quine says that the two tenets of empiricism are unassailable. One, the inculcation of the meanings of words (the conceptual) must ultimately rest on sensory evidence; two, whatever evidence there is for science is empirical (the doctrinal) (Quine 1969, 75). He contends that science—specifically, empirical psychology—explains how one acquires basic concepts, which serve

[1] See: (Putnam 1971, 73-4).

[2] There is a historical precedent for the pragmatists. G. Frege writes, "It is applicability alone which elevates arithmetic from a game to the rank of science" (Frege 1970, 187). Gödel held that view (Gödel 1990b, 269), and more recently, Maddy (Maddy 1992, 275). Also see: (Kitcher 1980, 219). Finally, P. Garden remarks, "[Jean Baptist Joseph Fourier (1768-1830)] was first and foremost a physicist, and he expressed very definitely his view that mathematics only justifies itself by the help it gives towards the solution of physical problems..." - from the introduction (Cantor 1918, 1). Brown defines applicability: "Mathematics hooks onto the world by providing representations in the form of structurally similar models" (Brown 1999, 49; also see 46-9).

[3] Logicism attempted to reduce mathematical concepts to logical ones, which was supposed to have a doctrinal pay-off. Similarly, for the logical positivists, natural knowledge was to be based on sense experience (Quine 1969, 71). Quine writes, "Just as mathematics is to be reduced to logic, or logic to set theory, so natural knowledge is to be based somehow on sense experience. This means explaining the notion of body in sensory terms; here is the conceptual side. And it means justifying our knowledge of truths of nature in sensory terms; here is the doctrinal side of the bifurcation" (Quine 1969, 71). Quine writes, "To endow the truths of nature with the full authority of immediate experience was as forlorn a hope as hoping to endow the truths of mathematics with the potential obviousness of elementary logic" (Quine 1969, 74). It was not that experimental implications were too complicated to trace. The problem was that large blocks of a theory may match sensory statements, but individual statements in the block may not (Quine 1969, 79).

as the foundation of knowledge.[4] There is still what is foundational (acquired, basic concepts) and what rests upon that (the doctrinal).

Utilizing science to explain the connection between evidence and knowledge seems to beg the question about the reliability of empiricism per se. Quine says the worry of circularity is annulled "once we have stopped dreaming of deducing science from observations" (Quine 1969, 76). As he puts it, "Better to discover how science is in fact developed and learned than to fabricate a fictious structure to a similar effect" (Quine 1969, 78). He goes on, "Epistemology, or something like it, simply falls into place as a chapter of psychology and hence natural science" (Quine 1969, 82).[5] His naturalized epistemology starts with science because it the best (i.e., the most successful) theory available (Quine 1969, 69-90; 1992, 19). According to the pragmatist, a reason that the scientific methodology produces successful. As Sellars explains:

> For empirical knowledge, like its sophisticated extension, science, is rational, not because it has a *foundation* but because it is a self-correcting enterprise which can put *any* claim in jeopardy, though not *all* at once. (Sellars 1997, 79)

Scepticism functions as part of the scientific methodology.

Quine's argument for the indispensability of abstract objects requires the following standard of ontological commitment:

> [A] theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory are true. (Quine 1953, 13)

When the terms of one's theory quantify over some objects, they must exist. As he writes:

> When we say, for example, (∃x)(x is a prime . x > 1,000,000), we are saying that *there is* something which is prime and exceeds a million; and any such entity is a number, hence a universal. In general, *entities of a given sort are assumed by a theory if and only if some of them must be counted among the values of the variables in order that the statements affirmed in the theory be true.* (Quine 1953, 103)

Quine is not, for example, advocating that when one tells the story of Cinderella, she must exist. He says that one must distinguish between explicitly presupposing X and not explicitly presupposing X (Quine 1953, 102). Quine writes, "What there is does not depend on one's use of language, but what one says there is does" (Quine 1953, 103).

Quine, also, is not slipping into some sort of linguistic or methodological idealism.[6] He writes:

> It is no wonder, then, that ontological controversy should end in controversy over language. But we must not jump to the conclusion that what there is depends on words. Translatability of a question into semantical terms is no indication that the question is linguistic. To see Naples is to bear a name which, when prefixed to the words 'see Naples', yields a true sentence; still there is nothing linguistic about seeing Naples. (Quine 1953, 16)

Numbers' existence is explicitly presupposed. As he writes, "For I deplore that facile line of thought according to which we may freely use abstract terms, in all the ways terms are used, without thereby acknowledging the existence of abstract objects" (Quine 1960, 119). For him, the existence of arithmetical objects is justified because they are quantified over in a theory, which is indispensable to scientific practice.[7]

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[4] It can be argued on behalf of the naturalized epistemologist, in the case of basic arithmetic for instance, the logic of discovery (e.g., as explained by P. Kitcher 1984).

[5] Just as one's eyes are irradiated in two dimensions and we see in three, similarly, concepts are used in constructing the world (Quine 1969, 84). Quine even suggests that some structural traits of colour perception - and induction itself - may have an evolutionary explanation (Quine 1969, 90). Also see: (Maddy 1990a, 620).

[6] Quine notes, for Dewey, knowledge, mind and meaning are part of the same world (Quine 1969, 26). Yet talk of a system, holism, and so on, can give the impression that Quine is lapsing into idealism. See, however, Quine's remarks: (Quine 1953, 16).

[7] Hacking has employed the phrase but my usage is completely independent of his.

Quine's argument for the indispensability of abstract entities is modeled upon the process by which one assents to the existence of objects in everyday life. According to him, humans assent to the existence of physical objects because they are basic to our language; the focus of successful communication and they allow for fairly direct conditioning (Quine 1960, 234, 238).

Physical and abstract objects seem to be on the same footing insofar as both are common to linguistic practices. But the suspicion, as he points out, is that physical objects are "better attested to" than abstract ones (Quine 1960, 234). Quine, however, notes that in order to assent to the existence of an object, one needs, first, comparative directness with sensory stimulation and second, utility for theory. For example, he says that when one points to a rabbit and announces "rabbit", a non-English speaker cannot know if we are referring to the rabbit or rabbit parts (Quine 1969, 46). This example of radical translation was supposed to show that sensory stimulations alone are not enough to know something. To pick out a rabbit requires a shared meaning embedded in language that supervenes upon sensory stimulations. One way to put it is that connotation is required for denotation. As Quine writes, "Talk of external things, our very notion of things, is just the conceptual apparatus that helps us to foresee and control the triggering of out sensory receptors" (Quine 1981, 1).

His argument for the indispensability of abstract objects depends upon his epistemological holism. Epistemological holism can be understood as the idea that theory helps in decisions about the acceptance and interpretation of data as much as data helps in choosing a theory (Quine 1981, 1). As he says, "Physical objects are postulated entities which round out and simplify the flux of experience just as the introduction of irrational numbers simplifies the laws of arithmetic" (Quine 1953, 19).

On Quine's analysis, the (epistemological) difference between physical and abstract objects is "illusory" (Quine 1981, 16). Empirical science is supposed to provide the explanatory bridge between how one gets from sensory stimulations to the recognition of objects (Quine 1981, 2, 22-3). According to Quine, the process by which one comes to know tables and chairs applies, more generally, to abstract arithmetical objects. As he puts it, ontology is an "outgrowth" of lay culture (Quine 1981, 9). He portrays our epistemic situation thus:

> The naturalistic philosopher begins his reasoning within the inherited world of theory as a going concern. He tentatively believes all of it, but believes also some unidentified portions are wrong. He tries to improve, clarify, and understand the system from within. He is the busy sailor adrift on Neurath's boat. (Quine 1981, 72)

Knowledge of abstract objects is merely a further extension of science. As Quine writes:

> At any rate the ontology of abstract objects is part of the ship which, in Neurath's figure, we are rebuilding at sea. (Quine 1953, 16)

> The ontology of abstract objects is part of the ship too. (Quine 1960, 124).

More specifically:

> [S]ince mathematics is an integral part of this higher myth, the utility of this myth for the physical sciences is evident enough. (Quine 1953, 18)[8]

> Epistemologically these [mathematical objects] are myths on the same footing with physical objects and gods, neither better or worse except for the difference in degree to which they expedite our dealings with sensory experiences. (Quine 1953, 45)

Arithmetical objects, according to Quine, fair no worse than everyday objects; they are all relative to our epistemological point of view, our "interests and purposes" (Quine 1953, 18-19). If one pictures knowledge as contained in a circle, one can say that the core contains solidified parts of the theory, which is indirectly assumed (stimulus-analytic statements), while the circumference is in contact with experience.

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[8] Quine writes: "The reason for admitting numbers as objects is precisely their efficacy in organizing and expediting the sciences. The reason for admitting classes is much the same" (Quine 1960, 237). In fact, he notes that mathematics did develop along side science (Quine 1981, 154).

Quine concedes, in fact, that there are degrees of closeness to experience. According to him, abstract objects exist because the language of arithmetic, for example, commits one to the numbers, which it quantifies over (Quine 1992, 30-1).[9] Mathematics, on his account of it, is empirical by its application in science (Quine 1992, 55). Decisions about what counts as real are made from within a theory, and this is supposed to be as true of physical objects as it is for abstract ones (Quine 1953, 102). Quine leaves decisions on what abstract objects to reify up to mathematicians and scientists (Quine 1960, 275).[10]

Stretched to its limits, it is Quine's holism that saves the unapplied parts of mathematics. (See §3.) The unapplied parts of mathematics are true because they are "couched in the same grammar and vocabulary that generate the applied parts of mathematics" (Quine 1992, 94).

The talk of epistemological holism, however, indicates anti-realism because proponents of that doctrine emphasize the idea that truth is linked to certain methods. Yet Quine avoids extreme conceptual relativity (any view of the world is as good as any other) and the idea that as one's methods change so does truth. As he writes, "'Truth' is one thing, warranted belief another. We can gain clarity and enjoy the sweet simplicity of two valued logic by heeding the distinction" (Quine 1992, 94). Suffice it to say that he is committed to the mind-independent existence of abstract objects like numbers.

2. Putnam

In the wake of Quine, it is useful to consider Putnam's views. First, Putnam, like Quine, is an anti-foundationalist. In "Mathematics Without Foundations" (1979a) Putnam expresses scepticism about foundationalist epistemology. Putnam's view is epitomized in his claim that when science and philosophy meet the latter changes, not the former (Putnam 1979a, 44). Science is more secure than epistemology (Putnam 1979a, 73). According to Putnam, one must begin with the truth of scientific and mathematical knowledge (Putnam 1979a, 11). Putnam writes:

> We will be justified in accepting classical propositional calculus or Peano number theory not because the relevant statements are 'unrevisable in principle', but because a great deal of science presupposes these statements, and because no real alternative is in the field. (Putnam 1998, 175)

According to Putnam, we have the right to take mathematics to be true because it is required for scientific practice.[11]

Second, however, Putnam's and Quine's employment of the indispensability argument differs in terms of their respective ontological commitments. Quine reifies mind-independent abstract objects and Putnam does not. Putnam accounts for mathematical necessity without platonism. According to him, the focus is upon "the truth of *p*" (not the mind-independent existence of numbers). Putnam gave up an earlier view where he held that mathematical entities are mind-independent (Putnam 1971).

Third, like Quine's efforts in the "Two Dogmas of Empiricism" (1953), Putnam criticizes the distinction between analytic-synthetic statements. He takes his departure from attacking the positivists' verificationist doctrine. Positivists, such as Carnap, had distinguished mathematical assertions from empirical ones, which are supposed to be verifiable by experience (Carnap 1935, 36).[12] Putnam rejects the idea that there is, on the one

[9] Quine writes: "But to view classes, numbers, and the rest in this instrumental way is not to deny having reified them; it is to explain why" (Quine 1981, 15).

[10] Quine writes: "Each reform is an adjustment of the scientific scheme, comparable to the introduction or repudiation of some category of elementary physical particles" (Quine 1960, 123). The same values that care for ontological economy in science apply to mathematics (Quine 1960, 269).

[11] Putnam remarks, the axiom of choice was defended because it was widely used (Putnam 1979a, 66-7, 76). Also see (Putnam 1978, 76; 1981, 73). Rather than viewing science as one block, Quine says, "more modest chunks suffice" (Quine 1981, 71). So, one can say some parts of knowledge are more closely tied together (say, within one domain), than the entire picture which may be not as closely conjoined.

[12] Mathematical statements, Carnap says, do not "possess any

hand, empirical knowledge and, on the other, the formal sciences (which are *a priori*) (Putnam 1979a, 1).

Putnam's attempt to follow practice results in an epistemologically egalitarian outlook. Ultimately all knowledge must be justified in the same way, that is, by describing how it is acquired. Moreover, the acquisition of knowledge is dependent on an entire world-view, which is just another way of saying that data is theory-laden (Putnam 1981, 215).[13]

Finally, like Quine, Putnam's epistemological holism—his internal realism—seems anti-realist. Truth and falsity function within different theories that provide various descriptions of the world. According to Putnam, there are different ways to conceive the world. As he writes, however, "But the question 'which kind of 'true' is really Truth' is one that internal realism rejects" (Putnam 1990, 96). Nonetheless he writes:

> There is only one possible explanation [why there is a great deal of epistemic consensus]: human interests, human salience, human cognitive processes, must have a structure, which is heavily determined by innate or constitutional factors. Human nature isn't all that plastic. (Putnam 1978, 56)[14]

Putnam's internal realism and his attack upon metaphysical realism are often seen as a radical break with his earlier, realist views (1979a). Suffice it to say that, the indispensability argument, and hence his early views, are consistent with aspects of internal realism.

What the pragmatists are up to can be explained thus. One can distinguish between two levels of justification. The first-level of justification concerns the content of a discipline.[15] At the first-level of justification an epistemological account must be faithful to practice. It would be peculiar to propose an epistemology completely at odds with epistemic practices.[16] (For example, if one proved mathematical theorems by flipping a coin, it could easily render false what would otherwise be true.)

The second-level of justification concerns the foundations of the first principles or methods required for practice. At the second-level, different types of justification for arithmetic, for example, both platonist and formalist, make "no difference to their [mathematician's] practice" (Maddy 1990, 3). That is, for example, whether one accepts the Peano axioms because they describe the nature of mind-independent numbers or as conventions makes no difference. What matters is that one accepts them.

According to H. Reichenbach, and more generally, epistemology has two features. One aspect requires describing how knowledge is acquired. The other aspect requires criticizing practice, i.e., laying out how it ought to be justified (Reichenbach 1938, 3). Reichenbach's distinction parallels a separation between the context of discovery from justification that goes back at least to Frege. According to Frege, one must separate how one discovers X (the descriptive task) from how one justifies X (the critical task) (Frege 1953, §3). Traditional foundational epistemologies, like Frege's, lead to the claim that

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factual content" (Carnap 1939, 2); for example, they do not yield any predictions as to be testable. Yet positivism suffers from several generic problems (arguably, from taking their thesis too far): (1) Claiming their division - the meaningless, analytic, and empirical - is semantic; rooting verification in a theory of meaning (statements about the past, future, present, and even concerning the external world once one dies) may be deemed vacuous (Reichenbach 1938, 73, 135). (2) They may have a simplified notion of science that excludes speculation as meaningless. And, (3), they abandon the realist hypothesis because it is empirically unverifiable. Finally, there own thesis seems empirically unverifiable, i.e., meaningless.

[13] For example, Putnam notes, inductive logics all depend on some *a priori* ordering of hypothesis (e.g. by simplicity). In mathematics the analogy would be, say, the axioms of set theory As he notes, in Newton's *Principia*, "rule 4" tells one that when there are two hypotheses one should choose the one that is accepted and *a priori* more plausible (Putnam 1979a, 66-7, 75).

[14] Putnam writes, "And I argued that being rational invokes having criteria of relevance as well as criteria of rational acceptability, and that all of our values are involved in our criteria of relevance. The decision that a picture of the world is true (or true by our present lights, or 'as true as anything is') and answers the relevant questions (as well as we are able to answer them) rests on and reveals our total system of value commitments. A being with no values would have no facts either" (Putnam 1981, 201).

[15] See Brown's discussion (Brown 1989, 133-151); he claims that history can be a guide to a normative methodology (Brown 1989, 151).

[16] In the case of mathematics, the first-level of justification "asks about the mathematics required to do science, the other asks about the foundational underpinnings of said mathematics" (Peressini 1997, 217).

describing how the Peano axioms are acquired is not to offer a justification for them. The descriptive and critical tasks must be separated for both levels of description.

Separating the context of discovery from that of the context of justification, however, may lead to problems (Sellars 1997, 13). First philosophy has often been described as leaving one with the problem of trying to pull oneself up by one's bootstraps. For instance, according to the foundationalist, the methodology that one takes to be the ground of knowledge cannot be justified by its employment. They cannot utilize method *P* to justify *P*. Traditionalists reject the notion that how one acquired the first principles can speak to their justification.

The pragmatists' solution is to reject the distinction between the context of discovery from justification at both levels of justification. They hold that there can be nothing deeper than describing how the methodologies one employs are acquired. They are motivated by a desire to escape foundational epistemology by emphasizing practice. Instead of trying to pull themselves up by their bootstraps, pragmatists explain how one bought the boots.

3. The Redemptive Function of Authoritarian Languages

It is worthwhile considering some classic problems with the indispensability argument, which has been raised by P. Maddy, a proponent of set-theoretic realism. Her criticisms must be refuted if the indispensability argument is to be a viable option. In offering replies to Maddy, I shall often show how they are inconsistent with her own project; I do not intend to take on each claim on its own, but highlight the high cost of endorsing her claims against the indispensability argument. Maddy asks, first, if there is such a thing as an accepted theory—science—which arithmetic could be indispensable to? (Maddy 1992, 280)

Yet Maddy assumes certain scientific standards of evidence (e.g., the causal constraint requirement) that she seeks to extend to sets. She cannot call into question the idea that there is such a thing as scientific standards of justification without undermining her set-theoretic realism. Her criticism is self-defeating.[17]

Second, according to Maddy, being useful is not equivalent to being true (Maddy 1992, 281). Mathematics' successful application in scientific inquiry may not warrant an epistemological pay-off. More generally, one cannot identify *P*'s success, with *P*'s truth. Maddy writes: "In short, legitimate choice of method in the foundations of set-theory does not seem to depend on physical facts in the way indispensability theory requires" (Maddy 1992, 289).[18]

Maddy contends, however, that mathematics' applicability is an argument against considering it a formal game (Maddy 1992, 275). According to her, mathematics' applicability is a reason to believe that one cannot make up mathematics anyway one wants. Mathematics' applicability is a reason to be a realist. She, therefore, must agree that being useful is a quality that is at least indicative of truth. Her criticism, once again, is at odds with what is required to motivate her set-theoretic realism.

Third, both Quine and Putnam must confront the issue of the unapplied parts of mathematics. As Maddy points out, justification in terms of abduction does not extend to the unapplied parts of mathematics (Maddy 1992, 278). She writes:

> The trouble is that this [the indispensability argument] does not square with the actual mathematical attitude towards unapplied mathematics...Here simple indispensability theory rejects accepted mathematical practices on non-mathematical grounds, thus ruling itself out as the desired philosophical account of mathematics as practised. (Maddy 1992, 278-9)

In addition, A. Peressini contends that one cannot know if the indispensability argument works even for the applied parts of mathematics until it is made clear what

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[17] See: (Quine 1969, 87; Putnam 1981 , 55; 1990, 178).

[18] Quine writes, "The fundamental use of natural numbers is in measuring classes; in saying that a class has 'n' members. Other serious uses prove to be reducible to this use" (Quine 1981, 15).

"units (theory, object, theorem, axiom)...what parts of pure mathematical theory are confirmed" (Peressini 1997, 216-7). He writes:

> It is not clear, however, that unitary operators have the same 'indispensable' status as do Hermitian operators, since unitary operators do not correspond to an aspect of physical reality in the way Hermitian operators do. (Peressini 1997, 221)

Quine can respond to these concerns. He clarifies an earlier view that would render the unapplied parts of mathematics unjustified. In recent writings, Quine casts the net wide, including the unapplied parts within the already utilized sections. According to Quine, non-applied parts of mathematics are true by inference. That is, if one part of mathematics is true (the terms it quantifies over exist), it is safe to say the rest is (the rest of the terms germane to the unapplied parts refer). As Peressini himself, notes, for Quine, mathematics get to be part of the boat, as it were (Peressini 1997, 226).

For Putnam, the non-applied parts of mathematics are true because they could be possibly applied (they could be applied for the same reason as Quine) (Putnam 1979a, 60). Putnam says, for example, that he regards sets of very high cardinality as "speculative and daring extensions of the basic mathematical apparatus of science" (Putnam 1979a, 56).

In addition to Maddy's criticisms, a number of more recent concerns have been raised with the indispensability argument and I shall consider them next. Fourth, as Peressini explains, for a pragmatist, scientific methodologies must be more secure than mathematical ones. After all, mathematics' principles can be justified by their employment in scientific theory (but not vice versa). As Peressini remarks, however, when scientific theories are discarded, the mathematics thus employed remains. The notion that scientific knowledge is more secure than mathematics is counter-intuitive and is therefore a reason to reject the indispensability argument (Peressini 1999, 258).

Yet, in the pragmatist's defence, it can be argued that we must distinguish between epistemological and metaphysical security. Even though mathematics is justified by way of its application in science that does not makes its truths less secure. Domains about which we are a realist have an equal claim to necessity. What differs between the two is the priority of justification—how one justifies one by way of the other. Yet the mathematical and scientific truths are, it can be argued, metaphysically on a similar footing in terms of the uniqueness of their respective bodies of knowledge.

Fifth, E. Sober, in an interesting attack on the indispensability argument, notes that mathematics has been used for a variety of theories, ones that are true and ones that are false (Sober 1993, 43, 55; Maddy 1997, 143). He explains, "It is less often noticed that mathematics allows us to construct theories that make *false* predictions and that we could not construct such predictively *unsuccessful* theories without mathematics" (Sober 1993, 53). Sober claims that mathematics is the background and cannot inherit support from the theories it participates in (Sober 1993, 53).

M. Colyvan, however, has responded to Sober by pointing out that since mathematics is not responsible for giving rise to false predictions or hypotheses, it can share in the credit for correct ones (Colyvan 1999, 330). Colyvan illustrates his point with an analogy:

> Blaming the mathematics is like a programmer blaming the programming language. And similarly, claiming that mathematics cannot share in the credit is like claiming that the programming language cannot share the credit for the successful program. (Colyvan 1999, 329)

One must distinguish between the fault of the scientist in coming up with a false theory, and the credit to be given to mathematics in the case of a successful scientific theory that utilizes mathematics.

Finally, and more generally, the merit of the indispensability argument has been undermined because of nominalists have also been able to account mathematics' applicability (e.g., Field 1989; Feferman 1998, 207). Yet, and in a nutshell, Field's program, for instance, is unacceptable until he can refute Shapiro (1997), and show

how his account can eliminate reference to abstract objects. Since the language of arithmetic in the twentieth century has been in terms of set theory, Feferman's alternative, for instance, is not constraining. There is no compelling reason to accept a nominalization. Indeed, the motivation for considering a nominalization is undercut, since, in the next section I provide a defence of a revised version of the indispensability argument that does not entail an over-blown ontology.

4. Revision

Thus far, the indispensability argument has been defended. Several further criticisms, by Peressini, however, require at least revising the indispensability argument, and they shall be considered next. If abduction is to apply generally it may allow for the countenancing of improper mathematical or scientific theories. We may want to note that sometimes, improper mathematical theories, for example, are also found useful in science:

> Newton introduced fluxions to perform the seemingly impossible role of being both zero and not zero: he had to assume that a variable was not zero in order to avoid dividing by zero, but then assumed that it was zero to get his results for instantaneous velocity...[Also, Dirac's delta function] was nonsense, and yet in spite of this it worked brilliantly in a successful physical theory [quantum mechanics]...These techniques actually 'fly in the face' of pure mathematics in the same way that dividing by zero 'flies in the face' of ordinary arithmetic. (Peressini 1997, 224)

Successful application of Newton's calculus would lead us to believe in the mind-independent existence of infinitesimals.

Peressini explains, in another criticism of indispensability, why the use of abduction in science differs from that in mathematics. He notes, "The physical application requires empirical bridge principles to underwrite physical interpretation" (Peressini, 1999, 214).[19] When deciding to assent to unobservables, for instance, rules of inference relating to some observations are necessary. Since physical objects causally function in explanations, a description of those objects are provided in scientific theories, but the same is not true, say, for the number three. Mathematics does not yield predictions ("124÷4=31" is not a prediction) nor (for Quine) do their entities stimulate our sensory receptors.

In fact, as Peressini points out, indispensability proponents who faithfully follow the analogy between science and mathematics must justify mathematics internally (as was the case in science) (Peressini 1999, 267). Numbers, for example, are often derived from premises not new observations (Field 1980, 10-11, 40). As Sober remarks, in a different context, therefore, "Indispensability is not a synonym for empirical confirmation, but its very antithesis" (Sober 1993, 44).

Justifying the existence of arithmetical objects in terms of abduction gives rise to the existence of mathematical objects that are epistemologically irrelevant.[20] The indispensability argument, in fact, would not lead one to know what mathematical objects are only that they exist (Feferman 1998, 297).

To avoid Peressini's attack, abduction should be reformulated thus:

> If our best scientific practice *q* presupposes *p*, then *q gives* one good reason to believe *p, only if the practitioners in the relevant field countenance p*.[21]

Scientists are likely to have a great deal of consensus over unobservables (eventually). Conversely, arithmetical objects would not fare well because consensus upon,

[19] Parson 1983, 195.

[20] B. Russell's words, written in a different context, express the doubt of inferring the existence of mind-independent abstract objects: "[The indispensability argument] has many advantages; there are the same as the advantages of theft over honest toil" (Russell 1919, 71). Benacerraf adds, "For with theft at least you come away with the loot, whereas implicit definition, conventional postulation, and their cousins [like the indispensability argument] are incapable of bringing truth" (Benacerraf 1973, 679).

[21] J. Azzouni's distinction between thin (e.g., mathematical) posits and thick (e.g., scientific) posits is another way of drawing the same line between when abduction should be utilized and when not (Azzouni 1994, 65).

for example, countenancing the mind-independent existence of numbers would likely not obtain.

The revised formulation provides a way to avoid assenting to the mind-independent existence of abstract objects. Furthermore, the revised formulation is faithful to Quine's intentions. Quine had claimed that decisions on what abstract objects to reify would be left up to practitioners (Quine 1960, 275). According to him, for example, practice is the final court of appeal. Quine, however, had also decided that numbers should be reified (Colyvan 1998, 50). Quine was not as faithful to practice as he could have been.

If we were to radically follow practice, only mathematicians would decide if numbers exist. Upon the revised formulation of abduction, arithmetical knowledge, for example, receives an internal justification since one follows the practice of mathematicians, not scientists or philosophers. Allowing mathematicians to be arbitrators, stands to immunize the indispensability argument from the charge of countenancing improper mathematics, and assenting to the mind-independent existence of abstract objects. Sure there will be matters of debate, as there are in all areas of science. Some will be worked out over time, and ontological ones are of no obvious consequence to mathematical knowledge in any case. Believe in the mind-independence of numbers if you will! For my part, I have argued we need not.

We may wish to recall that the problem of an over-blown ontology is a challenge, posed to Quine because he attempts to infer the existence of abstract objects. Putnam's tack, however, allows us to evade the problem of an over-blown ontology because he does not posit the existence of abstract objects. Following suit, I have revised the indispensability argument to exclude inferring the mind-independent existence of arithmetical objects, while still providing a basis for arithmetical realism. And, once again, the decisions about what counts as mathematics is something to be left to mathematicians.

At base of the pragmatist outlook is the idea that knowledge is developed to some end. In the case of the sciences generally, we have to put faith in the enterprise, its practitioners, to let science do the talking. It has become all too apparent that philosophers should not revise science, determine what exists, or construct elaborate stories of justification that are at odds with practice. Philosophers can still serve as social critics, historians of ideas, and perhaps offer clarity here and there; if they want to say something about already successful domains of knowledge, they should heed the pragmatist's tack, as I have attempted to.

In revising the indispensability argument I have attempted to immunize it from a number of key attacks, namely, that it is not clear what counts as mathematics and that pragmatists may commit us to an overblown ontology. Consolidating and revising the indispensability argument as it has been taken up by Quine and Putnam, I have reconfigured it to be more faithful to practice, and to shun commitment to the ontological reification of abstract mathematical objects.

At base, my argument is an articulation and defence of American common sense realism: if it works, it is likely true in some sense. In the case of the exact sciences, truth is, and must be, linked to our interests and pursuits, while at the same time being constrained by something beyond ourselves. The tension between construction and discovery is what I have tried to articulate within a realist, pragmatist framework. I have attempted to explain how the pragmatist can discuss "what works" in terms of "being true" for the exact sciences.

No easy feat, to be sure—that is good enough reason to have given it a shot. We need to continue to deepen our understanding of the socio-cultural context in which mathematics arises—the pragmatic impetus to developing mathematics—while at the same time being mindful of the seemingly mind-independence of mathematics.

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